

NOTATION

x , coordinate measured from the diaphragm along the shock tube channel, m; L , length, m; t , time measured from the time of diaphragm rupture, sec; T , temperature, °K; p , pressure bar; ρ , density, kg/m³; a , speed of sound, m/sec; u , stream velocity, m/sec; U , shock velocity, m/sec; μ , molecular weight, kg/kg · mole; M , stream or shock Mach number; γ , ratio of the specific heats; \bar{L} , dimensionless length, \bar{t} , dimensionless time.

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FLOW OF A VISCOUS TWO-TEMPERATURE NONEQUILIBRIUM IONIZED RADIATING GAS OVER BLUNT BODIES

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and V. F. Mymrin

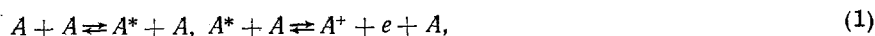
UDC 533.6.011

This paper investigates hypersonic flow of a monatomic viscous two-temperature nonequilibrium ionized radiating gas over blunt bodies. The transport coefficients are evaluated to a high-order approximation and their influence on the heat flux to the wall is analyzed.

An investigation of flow of a nonequilibrium ionized radiating gas over blunt bodies is a matter of great interest. The analogous problems were examined in [1-6] for a perfect gas, in [7, 8] (single-temperature approximation), and in [9, 10] (two-temperature approximation) for a viscous gas. However, it is suggested even in [9, 10] that the ionization reactions are frozen, radiation is absent, and the transport coefficients are calculated using very simple classical theory [11].

In this paper the problem of flow over a blunt body is posed in the most general form: the gas is regarded as viscous, heat-conducting, two-temperature, nonequilibrium-ionized, and radiating, and the transport coefficients are determined from high-order approximation theory.

The kinetic model of a gas (argon is chosen here) provides for atom-atom and electron-atom collisional ionizing reactions via an excited level



and also photon-ionization reactions with the ground level

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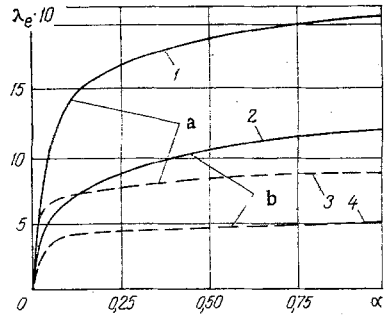


Fig. 1

Fig. 1. The electron thermal conductivity λ_e (J/m · sec · deg) in the third approximation (1, 2) and according to [11] (3, 4) as a function of the degree of ionization, for $p = 10^5$ N/m², $T_h = 2 \cdot 10^4$ °K and two values of T_e : a) $T_e = 16 \cdot 10^3$ °K; b) $12 \cdot 10^3$ °K.

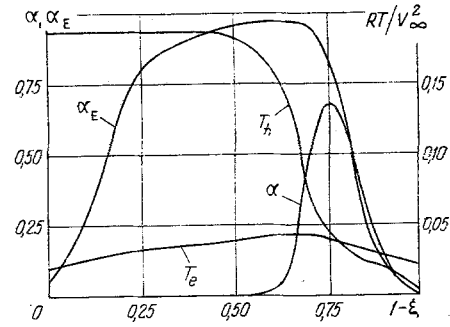


Fig. 2

Fig. 2. Profiles of nonequilibrium and equilibrium ionization, atom-ion temperature, and electron temperature of gases in the shock layer for $M_\infty = 30$, $p_\infty = 100$ N/m², $L = 0.04$ m, $\alpha_\infty = 10^{-3}$, $T_{h\infty} = 300$ °K, $T_{e\infty} = 10^4$ °K, $T_w = 2000$ °K (α , α_E , RT/V_∞^2 , and ξ are dimensionless quantities).



Expressions for the reaction rates for Eqs. (1)-(3) were given in [4, 7].

The initial system of equations includes the continuity equation, the energy equation for atom-ion and electron gases, the relaxation equation for the rate of ionization, and the radiative transfer equation [7, 12, 13].

To solve the problem we write these equations in a body-fixed coordinate system and transform them within the well-known thin shock-layer model [14]. Neglecting pressure variation across the layer, allowing for ambipolar diffusion and the presence of an internal electric field, and accounting for the radiative transfer equation in the plane slab approximation, we arrive at the following system of equations:

$$\frac{\partial}{\partial x} (r\rho u) + \frac{\partial}{\partial y} (r\rho v) = 0, \quad (4)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right), \quad (5)$$

$$\rho u \frac{\partial \alpha}{\partial x} + \rho v \frac{\partial \alpha}{\partial y} = m_a (\dot{n}_{aa} + \dot{n}_{ea} + \dot{n}_R) - \frac{\partial J_{iy}}{\partial y}, \quad (6)$$

$$\begin{aligned} \rho u \frac{\partial}{\partial x} \left(\frac{5}{2} RT_h \right) + \rho v \frac{\partial}{\partial y} \left(\frac{5}{2} RT_h \right) &= u \frac{\partial p_h}{\partial x} + v \frac{\partial p_h}{\partial y} \\ &+ \mu \left(\frac{\partial u}{\partial y} \right)^2 + n_e e E_y V_{iy} - \frac{\partial q_{hy}}{\partial y} - \frac{\partial q_R}{\partial y} - \omega_{ei} - \omega_{ea} \\ &- \xi_{aa} \dot{n}_{aa} - \xi_R \dot{n}_R - KT_j (\dot{n}_{aa} + \dot{n}_R), \end{aligned} \quad (7)$$

$$\begin{aligned} \rho u \frac{\partial}{\partial x} \left(\frac{5}{2} RT_e \alpha \right) + \rho v \frac{\partial}{\partial y} \left(\frac{5}{2} RT_e \alpha \right) &= u \frac{\partial p_e}{\partial x} + v \frac{\partial p_e}{\partial y} \\ &- n_e e E_y V_{iy} - \frac{\partial q_{ey}}{\partial y} + \omega_{ei} + \omega_{ea} - KT_j \dot{n}_{ea} + \xi_{aa} \dot{n}_{aa} + \xi_R \dot{n}_R, \end{aligned} \quad (8)$$

$$p = \rho R (T_h + \alpha T_e), \quad (9)$$

$$\cos \Theta \frac{dI_v}{dy} = \rho (1 - \alpha) \kappa_v (S_v - I_v), \quad (10)$$

where S_v is the source function,

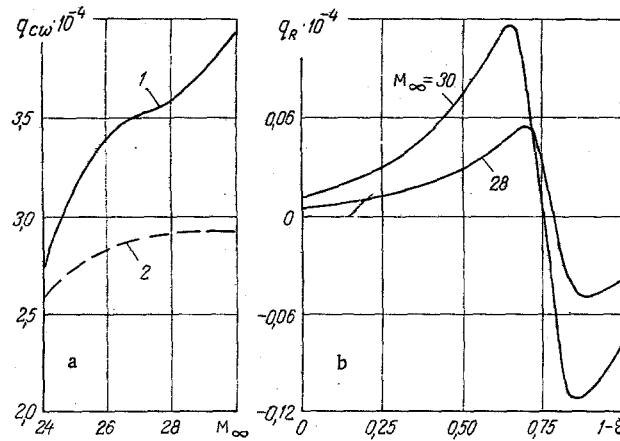


Fig. 3. The convective heat flux to the wall as a function of Mach number for the cases where the transport coefficients are calculated in higher-order approximations (1a), and according to [11] (2a), and the radiative heat flux distribution (b) in the shock layer. The flow conditions are $p_{\infty} = 100 \text{ N/m}^2$, $L = 0.04 \text{ m}$, $\alpha_{\infty} = 10^{-3}$, $T_{h\infty} = 300^\circ\text{K}$, $T_{e\infty} = 10^4\text{K}$, $T_w = 2000^\circ\text{K}$ (q_{cw} , q_R , kW/m^2 , M_{∞} , and ξ are dimensionless quantities).

$$S_v = \frac{\alpha^2}{1-\alpha} \cdot \frac{1-\alpha_E}{\alpha_E^2} B_v(T_e). \quad (11)$$

The boundary equations at the shock wave (we assume that the shock is a surface of discontinuity for the gasdynamic parameters) are the Rankine-Hugoniot relations, supplemented by the conditions for the degree of ionization $\alpha_{\infty} = \alpha_s$ and the electron temperature $T_{e\infty} = T_{es}$.

At the body surface the zero-slip conditions $u_w = 0$, $v_w = 0$ must be satisfied, and the heavy-component temperature can be placed equal to the wall temperature $T_{hw} = T_w$. The statement of the boundary conditions for the degree of ionization and the electron temperature must include formation of a wall layer of spatial positive charge. For a nonconducting wall and with no emission of electrons from the surface we have [9]

$$\left. \frac{d\alpha}{dy} \right|_w = \frac{1}{D_{Aw}} \left(\frac{KT_{ew}}{m_a} \right)^{1/2} \alpha_w, \quad (12)$$

$$\left. \frac{dT_e}{dy} \right|_w = \frac{1}{2} \left[\ln \left(\frac{m_a}{2\pi m_e} \right) - 1 \right] \frac{n_{iw} (KT_{ew})^{3/2}}{\lambda_{ew} m_a^{1/2}}. \quad (13)$$

In writing boundary conditions for the radiation we assume that the gas does not radiate, $I_{\nu s}^- = 0$ ahead of the shock wave, and that there is radiative energy balance at the body

$$I_{\nu w}^+ = \delta B_{\nu}(T_w) + (1-\delta) I_{\nu w}^-. \quad (14)$$

The expression for the spectral flux of radiative energy, obtained in the plane slab approximation for the case $\delta = 1$ and neglecting surface radiation from the body because of the low temperature, can be written as follows:

$$q_{R\nu} = 2\pi \left[\int_0^{\tau_{\nu}} S_{\nu} E_2(\tau_{\nu} - t_{\nu}) dt_{\nu} - \int_{\tau_{\nu}}^{\tau_{\nu s}} S_{\nu} E_2(t_{\nu} - \tau_{\nu}) dt_{\nu} \right], \quad (15)$$

where

$$E_2(z) = \int_1^{\infty} \omega^{-2} \exp(-\omega z) d\omega, \quad \tau_{\nu} = \int_0^y \rho (1-\alpha) \kappa_{\nu} dy, \quad \kappa_{\nu} = \sigma_j / m_a.$$

To calculate the total flux $q_R = \int_{\nu_j}^{\infty} q_{R\nu} d\nu$, we assume a multiband approximation for the absorption coefficient and use experimental values for the cross section for photoionization from the ground state $\sigma_j = 34 \cdot 10^{-18} \text{ cm}^2$ from the data of [15].

As a first step toward the solution in the entire subsonic region we consider the flow near the stagnation point. Here we assume that all the dependent variables, apart from the tangential velocity component $u = u_1(y)x$, the quantities $r = x$, and the pressure, determined by the Newtonian formula, are functions of a single variable y [14]. Then Eqs. (4)-(8) reduce to ordinary differential equations.

We introduce the variable η and express u_1 and v in terms of $f(\eta)$:

$$\eta = \left(\frac{V_\infty}{L\rho_s\mu_s} \right)^{1/2} \int_0^y \rho dy, \quad u_1 = \frac{V_\infty}{2L} f'(\eta), \quad v = -\frac{V_\infty}{L} f(\eta) \frac{dy}{d\eta}.$$

We now convert to dimensionless variables:

$$\begin{aligned} \bar{v} &= \frac{v}{V_\infty}, \quad \bar{u}_1 = \frac{u_1 L}{V_\infty}, \quad \bar{\rho} = \frac{\rho}{\rho_\infty}, \quad \bar{p} = \frac{p}{\rho_\infty V_\infty^2}, \quad \bar{T}_h = \frac{RT_h}{V_\infty^2}, \\ \bar{T}_e &= \frac{RT_e}{V_\infty^2}, \quad \bar{q}_R = \frac{q_R}{\rho_\infty V_\infty^3}, \quad \bar{\omega}_{ei} = \frac{L\omega_{ei}}{\rho_\infty V_\infty^3}, \quad \bar{\omega}_{ea} = \frac{L\omega_{ea}}{\rho_\infty V_\infty^3}, \\ \bar{\Phi}_{aa} &= \frac{Lm_a n_{aa}}{\rho_\infty V_\infty}, \quad \bar{\Phi}_{ea} = \frac{Lm_a n_{ea}}{\rho_\infty V_\infty}, \quad \bar{\Phi}_R = \frac{Lm_e n_R}{\rho_\infty V_\infty}, \\ \bar{\xi} &= \frac{\eta}{\eta_s}, \quad \bar{\varphi} = \frac{df}{d\eta}, \quad \tilde{\varphi}(\bar{\xi}) = \int_0^{\bar{\xi}} \varphi(\bar{\xi}) d\bar{\xi}. \end{aligned}$$

Then near the stagnation point Eqs. (4)-(8) take the form (we omit the bars above the dimensionless variables)

$$\frac{1}{\eta_s^2} \frac{d}{d\bar{\xi}} \left(l \frac{d\bar{\varphi}}{d\bar{\xi}} \right) + \tilde{\varphi} \frac{d\bar{\varphi}}{d\bar{\xi}} - \frac{1}{2} \bar{\varphi}^2 + \frac{4(1-k)}{\rho} = 0, \quad (16)$$

$$\frac{1}{\eta_s^2} \frac{d}{d\bar{\xi}} \left(\frac{l}{Sc} \frac{d\alpha}{d\bar{\xi}} \right) + \bar{\varphi} \frac{d\alpha}{d\bar{\xi}} + \frac{(\bar{\Phi}_{aa} + \bar{\Phi}_{ea} + \bar{\Phi}_R)}{\rho} = 0, \quad (17)$$

$$\begin{aligned} &\frac{1}{\eta_s^2} \frac{d}{d\bar{\xi}} \left(\frac{l}{Pr_h} \frac{dT_h}{d\bar{\xi}} \right) + \left[\frac{3}{5} \tilde{\varphi} - \frac{2}{5} \frac{l}{Sc} \frac{1}{\eta_s^2} \frac{1}{\alpha} \frac{d\alpha}{d\bar{\xi}} \right] \frac{dT_h}{d\bar{\xi}} \\ &- \frac{2}{5\rho} \left[\tilde{\varphi} \frac{d\rho}{d\bar{\xi}} + \frac{l}{Sc} \frac{1}{\eta_s^2} \frac{1}{\alpha} \frac{d\alpha}{d\bar{\xi}} \frac{d\rho}{d\bar{\xi}} \right] T_h - \frac{2}{5\rho} (\omega_{ei} + \omega_{ea}) \\ &- \frac{2}{5} \tilde{\varphi} (1) \frac{dq_R}{d\bar{\xi}} - \frac{1}{5} (2T_j + 3T_0) \frac{\bar{\Phi}_{aa}}{\rho} - \frac{2}{5} (T_j + T_e) \frac{\bar{\Phi}_R}{\rho} = 0, \quad (18) \end{aligned}$$

$$\begin{aligned} &\frac{1}{\eta_s^2} \frac{d}{d\bar{\xi}} \left(\frac{l}{Pr_e} \frac{dT_e}{d\bar{\xi}} \right) + \frac{3}{5} \left[\tilde{\varphi} \alpha + \frac{l}{Sc} \frac{1}{\eta_s^2} \frac{d\alpha}{d\bar{\xi}} \right] \frac{dT_e}{d\bar{\xi}} \\ &- \frac{2}{5} \left[\frac{5(\bar{\Phi}_{aa} + \bar{\Phi}_{ea} + \bar{\Phi}_R)}{2\rho} + \tilde{\varphi} \alpha \frac{1}{\rho} \frac{d\rho}{d\bar{\xi}} + \frac{l}{\alpha Sc} \frac{1}{\eta_s^2} \left(\frac{d\alpha}{d\bar{\xi}} \right)^2 \right. \\ &\quad \left. + \frac{l}{Sc} \frac{1}{\eta_s^2} \frac{1}{\rho} \frac{d\rho}{d\bar{\xi}} \frac{d\alpha}{d\bar{\xi}} + \tilde{\varphi} \frac{d\alpha}{d\bar{\xi}} - \frac{\bar{\Phi}_R}{\rho} \right] T_e + \frac{2}{5\rho} \\ &\quad \times \left(\omega_{ei} + \omega_{ea} - T_j \bar{\Phi}_{ea} + \frac{3}{2} T_0 \bar{\Phi}_{aa} \right) = 0. \quad (19) \end{aligned}$$

The corresponding boundary conditions can be written as

$$\bar{\xi} = 0, \quad \bar{\varphi} = 0, \quad \frac{d\alpha}{d\bar{\xi}} = \frac{Sc_w T_{ew}^{1/2}}{\tilde{\varphi}(1) \mu_w} \alpha_w,$$

$$T_h = T_w, \quad \frac{dT_e}{d\bar{\xi}} = \frac{1}{5} \left[\ln \left(\frac{m_a}{2\pi m_e} \right) - 1 \right] \frac{Pr_{ew} T_{ew}^{3/2}}{\tilde{\varphi}(1) \mu_w} d_w, \quad (20)$$

$$\bar{\xi} = 1, \quad \bar{\varphi} = 2, \quad \alpha = \alpha_s, \quad T_h = T_{hs}, \quad T_e = T_{es}. \quad (21)$$

Here

$$l = \rho\mu/\rho_s\mu_s, \quad Sc = \mu/\rho D_A, \quad Pr_h = 5R\mu/2\lambda_h, \quad Pr_e = 5R\mu/2\lambda_e.$$

In order to solve this system of equations we must know l , Sc , Pr_h , Pr_e , i. e., the transport coefficients as a function of the thermodynamic parameters of the gas.

It was shown in [19] that for ionized gases one must calculate these coefficients in higher-order approximations than are provided by the simplest classical theory. A method was proposed in [13] which makes it possible to calculate the transport coefficients in higher approximation for a two-temperature partially ionized gas, using a modified Chapman—Enskog method. Here a second-order approximation is sufficient for the viscosity coefficient, and a third-order approximation is sufficient for the electron thermal conductivity.

In this work we calculated the transport coefficients in higher approximations for the case of two-temperature nonequilibrium ionized argon. The interaction potentials of the various particle pairs were chosen as recommended in [16]. The investigation showed that our results for the general case agree well with the data of Devoto [16, 17] in the one-temperature approximation and with Spitzer and Harm [18] for a fully ionized gas.

It was found that, in general, the transport coefficients depend on the four independent parameters p , T_h , T_e , α .

Figure 1 shows results of calculating the electron thermal-conductivity coefficient. It can be seen that in the third approximation λ_e is considerably greater than λ_h , calculated using very simple classical theory [11].

Figures 2 and 3 show some results of the calculated flow over a body near the stagnation point. The solution was found by the sweep method, using double iteration on the BESM-4 computer. The gasdynamic field and the shock-layer standoff distance were determined during the internal iterations, and the radiation terms were improved in the external iterations.

Figure 2 shows the distribution of $\alpha(\xi)$, $\alpha_E(\xi)$, $T_h(\xi)$ and $T_e(\xi)$. It can be seen that the flow is appreciably nonequilibrium in most of the shock layer. Near the wall T_e exceeds T_h . A similar effect was noted in [9], where the special case of frozen flow at small Mach number was examined.

Figure 3 shows the convective heat flux q_{cW} to the body surface as a function of Mach number, and also the distribution of radiative heat flux q_R in the shock layer for $M_\infty = 28$ and 30. It can be seen that calculation of the transport coefficients in the higher approximations leads to an appreciable increase (up to 30%) in q_{cW} , compared with a calculation using transport coefficients derived from the simplest classical theory. By comparing Figs. 2 and 3b, we can see that the maximum of q_R lies in the cumulative ionization front region.

Investigations have shown that for the initial conditions considered the influence of radiation on the flow field is slight and the radiative flux to the body surface is less than the convective flux. However, as the Mach number increases, q_{RW} increases more rapidly than q_{cW} . The appreciable blocking of q_R by the wall region can also be seen.

NOTATION

A , A^* , atoms in the ground state and an excited state; A^+ , singly charged ion; x , y , coordinates; u , v , velocity components along the x and y axes, respectively; r , distance from the body axis of symmetry; ξ , dimensionless coordinate across the shock layer; V_∞ , M_∞ , gas velocity and Mach number in the incident stream; p , ρ , α , pressure, density, and degree of ionization of the gas; α_E , degree of equilibrium ionization of the gas; T_h , T_e , atom-ion and electron temperatures of the gas; V_i , diffusion velocity of the ions; J_i , mass flux of ions; E , internal electric field intensity; R , specific gas constant; K , Boltzmann constant; ν_j , T_j , ionization frequency and temperature; $B_\nu(T)$, Planck function; κ_ν , absorption coefficient for unit mass of an atomic gas; τ_ν , optical depth; σ_j , cross section for photoionization from the ground state; I_ν^+ , I_ν^- , spectral radiative intensities propagating in the positive (+) and negative (-) directions of the ξ axis; q_e , convective energy fluxes of atom-ion and electron gas; $q_{R\nu}$, q_R , spectral and total radiative energy fluxes; e , electron charge; m_a , m_e , masses of atom and electron; \dot{n}_{aa} , \dot{n}_{ea} , rate of ionization by atom-atom and electron-atom collisions; \dot{n}_R , rate of photoionization; ω_{ei} , ω_{ea} , rate of elastic energy exchange of electrons with ions and atoms; ξ_{aa} , ξ_R , mean energy of electrons formed by atomic ionization and photoionization; k , density rate

before and after the shock wave; μ , λ_n , λ_e , D_A , coefficients of viscosity, atom-ion and electron thermal conductivities, and ambipolar diffusion; δ , surface emissivity; n_i , ion concentration; Pr, Prandtl number; Sc, Schmidt number; $l = \rho\mu/\rho_s\mu_s$, dimensionless parameter; h, e, subscripts referring to parameters of the atom-ion and the electron gas, respectively; ∞ , s, w, subscripts referring to parameters of the gas in the incident flow, immediately behind the shock wave, and at the body, respectively.

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A CLASS OF MULTIPLE INTEGRALS OF TRANSFER THEORY

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UDC 539.125.523

We consider a class of multiple integrals of transfer theory under the assumption that the scattering field function may be exponential.

The scattering amplitude of two particles is determined by the Lippmann-Schwinger integral equation [1]

$$t(\mathbf{k}, \mathbf{k}', E) = V(\mathbf{k}, \mathbf{k}') + \int_{\Omega_1} \frac{V(\mathbf{k}, \mathbf{p})t(\mathbf{p}, \mathbf{k}', E)}{E - p^2 + i0} d\mathbf{p}, \quad (1)$$

where

$$V(\mathbf{k}, \mathbf{k}') = \frac{1}{(2\pi)^3} \int_{\Omega_2} \exp[-i(\mathbf{k} - \mathbf{k}')\mathbf{r}] V(\mathbf{r}) d\mathbf{r};$$

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